Contents

Part I Characterization

1 From Complex to Spatial Networks 3
1.1 Early Days 3
1.2 Complex Networks 4
1.3 Space Matters 5
1.4 Definition and Representations 5
1.4.1 Spatial Networks 5
1.4.2 Representations of Networks 6
References 8

2 Planar Graphs 9
2.1 Graph Theoretical Tools 10
2.1.1 Definition and Representation 10
2.1.2 K_5 Is Not Planar 11
2.1.3 Euler's Formula 12
2.1.4 Distance from Planarity 13
2.2 Planarity of Street Networks 18
References 20

3 Directed and Mixed Graphs 23
3.1 Theoretical Results 23
3.1.1 Definitions 23
3.1.2 Components of a Digraph 24
3.1.3 Statistics of Loops 25
3.1.4 Perturbation of the Shortest Path in a Directed Lattice 27
3.2 Empirical Results: One-Way Streets 28
3.2.1 Fraction of One-Ways 30
3.2.2 Statistics of the Detour Index 31
3.2.3 Asymmetry of Shortest Paths 33
References 36

4 Simple Measures 39
4.1 Irrelevant Measures for Spatial Networks 39
4.1.1 Degree 40
4.1.2 Length of Segments 42
4.1.3 Cell Area 44
4.1.4 Clustering and Assortativity 45
4.1.5 Average Shortest Path and Diameter 47
4.1.6 Empirical Illustrations 48
4.2 Simple Characterizations 54
4.2.1 α and γ Indices 54
4.2.2 Organic Ratio and Ringness 55
4.2.3 Edge Orientation Distribution 56
4.2.4 Shape Factor 58
4.2.5 Detour Index (or Stretch or Route Factor) 58
4.2.6 Cost, Efficiency and Robustness 60
References 63

5 Betweenness Centrality 65
5.1 Definition 66
5.2 General Properties 66
5.2.1 Numerical Calculation: Brandes' Algorithm 67
5.2.2 The Average BC 68
5.2.3 Edge Versus Node BC 69
5.2.4 Adding Edges 70
5.2.5 Relation Between the BC and the Clustering Coefficient 72
5.2.6 Scaling of the Maximum BC 72
5.3 The BC for Simple Graphs 74
5.3.1 Regular 1d and 2d Lattices 74
5.3.2 Cayley Tree 77
5.3.3 Branches and Ring 78
5.3.4 Grid-Tree Network 83
5.4 The BC in a Disk: The Continuous Limit 87
5.4.1 The Infinite Density Limit 87
5.4.2 Finite Density: A Perturbation Expansion 90
5.5 Empirical Measures on Street Networks 97
5.5.1 Spatial Patterns of Large BC Nodes 97
5.5.2 The BC and Socio-Economic Indicators 101
5.5.3 The BC Probability Distribution for Street Networks 101
5.5.4 The Effect of One-Way Streets on the BC 105
References 107

6 The Shape of Shortest Paths 109
6.1 (Euclidean) First Passage Percolation 109
6.1.1 Models and Definitions 109
6.1.2 Known Results About Exponents 112
6.1.3 Numerical Results 115
6.1.4 Travel Time and Transversal Fluctuations 117
References 118

7 Simplicity and Entropy 121
7.1 Simplicity 121
7.1.1 Simplest Paths 121
7.1.2 The Simplicity Index and the Simplicity Profile 124
7.1.3 A Null Model 125
7.1.4 Measures on Real-World Networks 125
7.2 Information Perspective 130
7.2.1 Entropy and Simplest Paths 131
7.2.2 Quantifying the Complexity 133
References 137

8 Large-Scale Tools 139
8.1 Spatial Dominance 140
8.1.1 Constructing the Dominance Tree 140
8.2 Class First-Passage Times 144
8.3 Community Detection in Spatial Networks 146
8.3.1 Modularity 147
8.3.2 A Null Model for Spatial Networks with Marks 148
8.3.3 Synthetic Spatial Network Benchmarks 152
8.3.4 Modifying the Modularity 153
8.4 Persistent Homology 157
8.4.1 Topological Data Analysis 157
8.4.2 Empirical Results 159
References 165

9 Typology of Planar Graphs 167
9.1 Area and Shape of Faces 167
9.1.1 Characterizing Blocks 168
9.1.2 A Typology of Planar Graphs 171
9.2 Approximate Mapping of a Planar Graph to a Tree 174
9.3 An Exact Bijection Between a Planar Graph and a Tree 178
9.4 Machine Learning Approaches 182
References 184

10 Measuring the Time Evolution of Spatial Networks 187
10.1 Road Networks 188
10.1.1 Organic Growth 188
10.1.2 Effect of Planning 196
10.1.3 Simplicity Measures 204

10.2 Subways . . . 208
10.2.1 Generalities . . . 208
10.2.2 Typical Numbers . . . 210
10.2.3 Network Evolution . . . 212
10.2.4 Standard Measures . . . 213
10.2.5 Efficiency . . . 214
10.2.6 Temporal Statistics: Bursts . . . 217
10.2.7 Core and Branches: Measures and Model . . . 219
10.2.8 Spatial Organization of the Core and Branches . . . 227
References . . . 231

Part II Models

11 Spatial Generalizations of Random Graphs . . . 235
11.1 Spatial Version of Erdos-Renyi Graphs . . . 235
11.1.1 The Erdos-Renyi Graph . . . 235
11.1.2 Planar Erdos-Renyi Graphs . . . 237
11.2 The Hidden Variable Model for Spatial Networks . . . 238
11.2.1 Effect of Space . . . 239
11.2.2 Effect of Traffic . . . 240
References . . . 241

12 Spatial Small-Worlds . . . 243
12.1 The Watts-Strogatz Model . . . 243
12.2 Spatial Generalizations in Dimension d . . . 244
12.3 Navigability in the Kleinberg Model . . . 247
12.3.1 Searchable Networks . . . 247
12.3.2 Sketch of Kleinberg's Proof . . . 249
References . . . 252

13 Growing Spatial Networks . . . 253
13.1 Preferential Attachment and Space . . . 253
13.1.1 Preferential Attachment and Distance Selection . . . 255
13.1.2 Searching in Spatial Scale-Free Networks . . . 262
13.2 Attraction Potential Models . . . 263
13.2.1 The Connection Rule . . . 264
13.2.2 Uniform Distribution of Nodes . . . 265
13.2.3 Exponential Distribution of Centers . . . 266
13.2.4 Effect of Centrality and Density . . . 267
13.2.5 The Appearance of Core Districts . . . 275
References . . . 276

14 Tessellations of the Plane . . . 277
14.1 The Voronoi Tessellation . . . 277
14.1.1 Average Properties of the Poisson-Voronoi Tessellation . . . 279
14.1.2 Statistical Properties . . . 281

14.1.3 Central Limit Theorem for the Total Length 285
14.2 Effect of the Density of Points 286
14.3 Crack and STIT Tessellations 288
14.4 Planar Fragmentation 289
References 292

15 Proximity Graphs 295
15.1 Random Geometric Graphs 296
15.1.1 The Hard Case 296
15.1.2 Soft Random Geometric Graphs 301
15.1.3 The Full Connectivity Probability 302
15.1.4 The Waxman Model 304
15.1.5 Random Geometric Graphs in Hyperbolic Space 308
15.2 Bluetooth Graph and Sparsification 309
15.3 The $k-$nearest Neighbour Model 310
15.3.1 Definition and Connectivity Properties 310
15.3.2 A Scale-Free Network on a Lattice 311
15.4 A Dynamical Proximity Model 313
15.4.1 The Model 313
15.4.2 Stationary State 314
15.4.3 Percolation Properties 315
15.4.4 Degree Distribution 315
15.5 Apollonian Networks 316
References 317

16 Excluded Volume Graphs 319
16.1 Delaunay Graph 319
16.2 Gabriel Graph 320
16.3 Relative Neighborhood Graph 323
16.4 β-Skeletons 323
References 326

17 Loops and Branches 327
17.1 Reducing the Complexity of a Spatial Network 327
17.2 A Loop and Branches Toy Model 330
17.2.1 Exact and Approximate Formulas for the BC 331
17.2.2 Threshold Value of w and Optimal ℓ 335
17.3 Analyzing the Impact of Congestion Cost 339
17.3.1 An Exactly Solvable Hub-and-Spoke Model 339
17.3.2 Congestion and Centralized Organization 343
References 346

18 Optimal Networks 347
18.1 Optimization, Complexity, and Efficiency 347
18.1.1 Complexity 347
18.1.2 Efficiency of Transport Network 348
18.2 Minimum Spanning Tree 350

18.2.1 Minimum Spanning Tree on a Complete Graph 351
18.2.2 Properties of the Euclidean Minimum Spanning Tree 353
18.3 Geometric t-Spanners 359
18.3.1 Definition 359
18.3.2 The Theta Graph 360
18.4 Optimal Trees: Generalization 361
18.5 Beyond Optimal Trees: Noise and Loops 366
References 369

19 Optimal Transportation Networks and Network Design 373
19.1 Empirical Motivation: The Structure of Subway Networks 374
19.2 Hub-and-Spoke Structure 376
19.3 One-Dimensional Problems 379
19.3.1 A Single Open Line 379
19.3.2 Transition for a Cyclic Service Line 383
19.4 Multiple Transit Lines 386
19.4.1 Parallel Transit Lines 386
19.4.2 Radial Lines 388
19.5 The Optimal Subway Problem 392
19.5.1 A First Simplification: Optimal Placement 393
19.5.2 The Minimum Average Distance to the Center 397
19.5.3 Average Minimum Time Between All Pairs of Points 400
References 405

20 Greedy Models 407
20.1 A Model for Distribution Networks 408
20.2 Cost-Benefit Analysis 410
20.2.1 Theoretical Formulation 411
20.2.2 Crossover Between the Star Graph and the MST 412
20.2.3 Spatial Hierarchy and Scaling 415
20.2.4 Understanding the Scaling with a Toy Model 418
20.2.5 Efficiency 419
20.3 Cost-Benefit Analysis: General Scaling Theory 422
20.3.1 Theoretical Framework 424
20.3.2 Subways 425
20.3.3 Railways 429
References 433

Index 435