Identifying city boundaries
Monofractal and multifractal approaches

Quenturb seminar
Institut des systèmes complexes, Paris
6 mai 2015
MAUP: Modifiable Areal Unit Problem
Openshaw (1983)

Case studies in geography are strongly dependent on the size of the spatial units used for the analysis.

Analysing the MAUP
Fractal approach = focus on the variation of the spatial distribution of a phenomenon across scales.
Phenomena under consideration

**Human settlements** (buildings, road networks, natural areas...) = the container for human activities

Geographic features under study: shapes, morphologies (urban and non urban)

**Human activities** (living, producing, exchanging, moving) = the content of human settlements

Geographic features under study: activity locations, trip patterns, migration flows
Phenomena under consideration

The container: buildings, road networks, natural areas...
=> Space

Both the container and the content (human activities)
=> Place

Set of places: urban region, urban agglomeration
Space and place configurations: concentration or dispersion

Examples
- Compact or sprawled built patterns
- Concentrated or dispersed migration flows
- High or low densities of population

Multiscale analysis of these configurations

Several levels of aggregation (Grasland 2003)

and/or

A series of spatial resolutions
(Goodchild & Mark 1987; Lam & Quattrochi 1992)

and/or

Analysis ranges (neighbourhoods) of different sizes
(van Vliet et al. 2009; White 2005)
**Research purpose:** identifying city boundaries

Interest discussed in a previous Quanturb seminar

**Results**

**Main Findings**

> Many urban attributes scaling depends on **city definition**

> Definition criteria do not affect all **attributes** in the same way and magnitude.

> The **population criteria** appears decisive to make scaling results vary.

... it’s also the easiest to harmonize for comparison
FIRST PART
A fractal analysis of theoretical cities and Belgian cities

Cécile Tannier and Isabelle Thomas

MAUP:
In case of fractal built patterns, the built density is reduced when passing from a given spatial resolution to a finer one
(Batty & Kim 1992) (Thomas et al. 2007)
Identifying city boundaries

Density $\rho_i = 5/9 = 0.56$

Density $\rho_{i+1} = 25/81 = 0.309$

Fractal dimension

$$\frac{\rho_{i+1}}{\rho_i} = \left(\frac{1}{r^2}\right) N = (r)^{(2-D)}$$

Reduction factor

Number of built elements
Our application

Assumption: urban built patterns are fractal. Hence their built density varies across scales.

A crucial change in the variation of built density from one scale to another one = A major morphological discontinuity

This crucial discontinuity defines the limit between two subsets of built elements: urban and non urban
Source data: vector maps representing buildings in two dimensions (polygons)
Morphological criteria: several advantages

They are quite objective and comparable

They represent the most enduring part of urban settlements

They can be build using parsimonious data
Morphological delineation of cities: usual approach

Classification of remotely-sensed images

Contiguity criterion: distance threshold defined *a priori*

Difficult to deal with urban-rural fringes

(a) The northern fringe of the city of Besançon (East of France)

(b) The western part of the urban area of Lille (North of France)
Covering

(a) Initial pattern: step 0

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Covering

Initial pattern: step 0

Covering the pattern: step 1
Covering

Initial pattern: step 0

Covering the pattern: step 1

Covering the pattern: step 2

$\text{zoom}$
Covering

Identifying city boundaries

Covering the pattern: step 1

\[ N = \left( \frac{I}{s} \right)^{-D} \]

\[ \frac{N_2}{N_1} = N \quad \text{and} \quad \frac{L_2}{L_1} = s \]

\[ L_2 \]

\[ \frac{L_1}{l_2} \]

zoom
Covering
Initial pattern: step 0

Covering the pattern: step 1

Covering the pattern: step 2

Covering the pattern: step 3
Covering

Initial pattern: step 0

Covering the pattern: step 1

Covering the pattern: step 2

Covering the pattern: step 3

morphological agglomeration
The chosen method: dilation instead of covering

(Minkowski 1903)
Theoretical fractal patterns: the number of built clusters is related to the size of the dilation buffer according to a hyperbolic function.
The dilation curve in the case of regular fractal patterns:

→ the number of built clusters decreases in the course of dilations according to a power law

\[ N = \varepsilon^{-D} \]

→ the curve is a straight line on a log-log plot

→ no significant threshold (no breaking in the curve)
The dilation curve may also not be a straight line
Fractal delineation of urban agglomeration

Three-steps methodology
1. Step-by-step dilation and counting the number of built clusters

Dilation curve

Number of built-up clusters

Width of the dilation buffer (in meters)

Buffer 50m => 64 clusters
Buffer 100m => 30 clusters
Buffer 250m => 1 cluster
2. Identification of a significant threshold on the dilation curve
2. Identification of a significant threshold on the dilation curve

- Dilation curve: Number of built-up clusters vs. Width of the dilation buffer (in meters)
- BIC for polynomial estimations of the dilation curve: Value of the BIC vs. Polynomial degree of estimated curves
- Estimated curve: Number of built-up clusters vs. Width of the dilation buffer (in meters)

Curvature function of the chosen estimated curve:

Curvature value vs. Width of the dilation buffer (in meters)
Curvature

→ Method widely used in image analysis (Lowe, 1989)

\[ \kappa = \frac{y''}{(1 + y'^2)^{3/2}} \]

→ Maximum curvature:
maximum deviation with regard to a linear decrease in the number of built clusters in the course of dilation

The curvature of a straight line is always zero.
The curvature of a circle is constant.
The calculation of the curvature function of a discrete curve shows every local variations

→ We have to fit the empirical dilation curve (discrete) with a polynomial (continuous) before calculating the curvature
Curvature

Polynomial of 7th degree

Curvature function

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Curvature

Polynomial of 7\textsuperscript{th} degree

Curvature function

Main curvature (300m)
Look at the polynomial estimation of the dilation curve...
2. Identification of a significant threshold on the dilation curve

- Dilation curve
- BIC for polynomial estimations of the dilation curve
- Estimated curve

Curvature function of the chosen estimated curve
Software: Morpholim

Developed by G. Vuidel (ThéMA, Besançon)

Available on Sourceforge

Inputs/outputs: shapefiles
3. Rank-size distribution of delineated built clusters

a- Sierpinski carpet

b- Fournier dust

c- Hybrid fractal carpet

d- Compact city

e- Non hierarchical polycentric city

f- Dispersed city
3. Rank-size distribution of delineated built clusters
4. Mapping the urban boundary

- Sierpinski carpet

- Fourier dust

- Non-hierarchical polycentric city

- Hybrid fractal carpet

- Dispersed city

- Compact city

- Sierpinski Carpet City

- Fourier Dust City

- Hybrid Fractal City

- Non Hierarchical Polycentric City

- Dispersed City
Application on six French and Belgian urban areas

Number of built-up clusters

<table>
<thead>
<tr>
<th>Urban area</th>
<th>Distance threshold (in meters)</th>
<th>Main curvature value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belfort</td>
<td>170</td>
<td>0.36</td>
</tr>
<tr>
<td>Besançon</td>
<td>276</td>
<td>0.43</td>
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<tr>
<td>Montbéliard</td>
<td>354</td>
<td>0.38</td>
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<tr>
<td>Namur</td>
<td>465</td>
<td>0.23</td>
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<tr>
<td>Charleroi</td>
<td>236</td>
<td>0.22</td>
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<tr>
<td>Liège</td>
<td>207</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Application on the 18 largest Belgian cities
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Classification 5 morphological criteria
\( D_B(MA), \delta, CO, \%B(MA), \) density

18 Belgian cities + 4 theoretical cities
Identifying city boundaries

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SECOND PART
Multifractal analysis of the spatial distribution of population

François Sémécurbe and Cécile Tannier

MAUP:
Population density varies greatly with the spatial resolution of the space partition under consideration

Objective
Quantifying the MAUP and using this quantification for describing the settlement pattern under consideration
Container: grid cells of 200 m width

Content: number of inhabitants in each grid cell

Source data: 200m-grid population 2010 dataset, French National Institute of Statistics and Economic Studies (INSEE)

Multifractal analysis:
- a series of nested spatial resolutions
  +
- a series of points of view about the quantity of information contained in each spatial unit
Multifractal formalism

Generalized dimensions $D_q$ (or $q$–dimensions)

$$D_q = \lim_{\delta \to 0} \frac{\frac{1}{1-q} \log \left( \sum_{i=1}^{n} \mu_{\delta,i}^q \right)}{-\log(\delta)}$$

$\mu_{\delta,i}$ is merely the population contained in cell $i$

The numerator is the generalized entropy defined by Renyi (Appleby 1996).

When $q$ tends to 1, the generalized dimension $D_1$ converges on the information dimension.

When $q = 0$, the generalized dimension $D_0$ is the fractal box-counting dimension of the support of the measure $\mu_{\delta,i}$.

When $q = 2$, the generalized dimension $D_2$ is called the correlation dimension $\Rightarrow$ probability that two individuals selected randomly will be a distance $\delta$ apart (Seuront 2009).
Multifractal formalism

Multifractal spectrum

\[ \alpha(q) = \lim_{\delta \to 0} \sum_{i=1}^{n} \mu_q(\delta, i) \frac{\log(\mu_q(\delta, i))}{-\log(\delta)} \]

\[ f(\alpha(q)) = \lim_{\delta \to 0} \sum_{i=1}^{n} \mu_q(\delta, i) \frac{\log(\mu_q(\delta, i))}{-\log(\delta)} \]

\[ 0 \leq f(\alpha) \leq \alpha \]

The intensity of local singularities constrains their spatial distribution: a very singular behavior cannot occur in too many places.
Multifractal parameters estimation

992 square spatial units regularly dividing the area of mainland France
Multifractal parameters estimation

A square spatial unit, \( \delta \)-grid cells, and 200m-grid population cells
Multifractal parameters estimation

Choice of the scale range of $\delta$ values for estimating multifractal indexes: between 800m and 6400m

Choice of the size of the square spatial units $\zeta$: 25km side length

Selection of a set of $q$ values: $\{0, 0.5, 1, 1.5, 2\}$
Classification of multifractal spectra

When the multifractal spectrum concentrates on small $\alpha$ values, the population is locally concentrated whatever the spatial resolution under consideration.

When the multifractal spectrum concentrates around 2, the population is spatially uniformly distributed: densities are almost constant locally.
Map of the spatial units classified according to their multifractal spectrum
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Map of the spatial units classified according to their multifractal spectrum

Figure 5: Delineation of French *aires urbaines* (metropolitan areas). In France, an *aire urbaine* encompasses a densely built *unité urbaine* - which is akin to a U.S. urban area - and its commuter belt. *Source: French National Institute of Statistics and Economic Studies (INSEE) 2010*
Conclusion of part II

Urban sprawl did not shape French periurban settlement patterns in a single way.

Pre-existing rural settlement patterns (openfield landscapes and enclosed field landscapes) have shaped patterns of urban sprawl locally.
General conclusion

Two implications for planning:
1- it would be useful to adjust the urban delineations to local specificities instead of applying systematically predefined delineation criteria
2- it does not seem possible to define general planning solutions, which could be applied successfully in any location

Perspectives

Compare the city delineation obtained by a fractal analysis of built patterns with the delineation obtained by a multifractal analysis of aerial photographs